A Machine Learning Factor-Based Interpretation for the Bond Risk Premia in the U.S.

Caio Vigo Pereira

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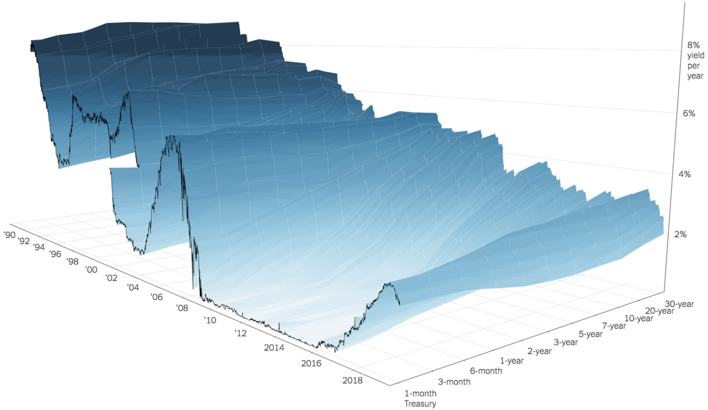
Motivation

- I study the time variation of the **risk premia in U.S. Treasuries bonds**.

- Treasury bonds play an important role in financial markets $\Rightarrow$ its risk and return dynamics is of **central economic importance**.
  - ↳ major importance for monetary policy
  - ↳ strategic in investors' portfolios
  - ↳ understanding of financial events: e.g., zero rates in 2008, 2020

- Understanding pricing of U.S. Treasuries is a central question in the study of bond markets.
  - ↳ The U.S. Treasury market is the largest government debt market in the world with an estimated value of **$14 trillion (2019)**.
  - ↳ $\approx$ **30%** of the entire U.S. bond market (corporate debt + mortgage and municipal bonds + money market instruments + asset-backed securities)

Treasuries Yields



risk premia: *difference between the current long rate and the expected average of future short rates.*

Motivation

- **Bond Premia**
  ↳ Long literature. Back from Fama and Bliss (1987)
  ↳ Nonetheless, never fully answered/understood
  ↳ Many **factors** were proposed in the literature:
    ↳ Fama and Bliss (1987) → *forward spreads*
    ↳ Cochrane and Piazzesi (2005) → *linear combination of forward rates*
    ↳ Ludvigson and Ng (2009) → *linear combination of macro PC loadings*
    ↳ Cieslak and Povala (2015) and Lee (2018) → *trend inflation*

Bauer and Hamilton (2018)

↳ evidence against the use factors not derived from the yield curve (non spanning) → **"spanning puzzle" literature**

↳ raised methodological issues: econometric problems when overlapping returns is used.

Research Question

- An important question that could assist to elucidate the whole bond premia problem is related with the factor structure of expected returns. **Is there a factor representation? If so, what is its structure?**

- Recently, Cochrane (2015) argued that it is possible that there is a dominant single factor structure for bond returns, in such a way that risk premiums rise and fall together.
  ↳ *A **parsimonious** number is key here.*

Central Question

- *What is the linear combination of forecasting variables that captures common movement in expected returns across assets?*

Introduction

A Different Route

It is possible that this search for deriving, building and estimating factors that represent state variables in macro-finance models may be limited.

- The process done by financial economists of manually discovering and hand picking this list of factors may be leaving unseen relationships between the state variables out in their derivation.

- To do so, I make use of one of the most powerful approaches in machine learning: **deep neural network** to uncover relationships in the full set of information from the yield curve.

Contribution

Methodological/Theory

- I propose a novel approach for deriving a **parsimonious number** of state factor consistent with a **dynamic term-structure with unspanned risks** theoretically motivated model.

- I use **deep neural networks** to uncover relationships in the full set of information from the yield curve, I derive a single state variable factor that provide a better approximation to the spanned space of all the information from the term-structure.

- I also introduce a way to obtain **unspanned risks** from the yield curve that is used to complete the state space.

Empirical Findings

- I show that this parsimonious number of state variables have predictive power for excess returns of bonds over 1-month holding period (in/out-of-sample).

- I provide an intuitive interpretation of derived factors, and show what information from macroeconomic variables and sentiment-based measures they can capture.

Contribution - **Discussion**

- **First,** through DNNs, we can introduce **nonlinearities** when modeling the bond risk premia in our first step of the recursive process.
  ↳ while still making use of a linear combination of the latent factors in the second step,
  ↳ and generating a parsimonious number of factors (state variables).

  ↳ With neural networks we can introduce flexible and complex nonlinear relationships from the inputs while approximating arbitrarily well a rich set of smooth functions.

  ↳ Consistent with recent findings (e.g., Gu et al. (2018); Bianchi et al. (2019)) → importance of allowing for **nonlinearities**.

  ↳ The approach is at the intersection of bond premia and **sequential learning** as in Gargano et al. (2019) and Dubiel-Teleszynski et al. (2019).

- **Second,** the approach avoids hand-picking the variables from the yield curve
  ↳ as through a DNN we are able to recursively learn the best-approximating function that condenses the yield curve into a single latent factor.

Contribution - **Discussion**

- **Third,** I overcome some the issues raised by Bauer and Hamilton (2018)
  ↳ use of non-overlapping returns, as done by the most recent literature (Gargano et al., 2019)
  ↳ I make use of the term structure at the higher frequency of 1-month holding period with maturities ranging up to 60 months ahead.

- **Fourth,** we start our process with only information from the term structure.

- **Fifth,** we take a broader interpretation of the unspanning factor.
  ↳ we can link with other sources of risks (macroeconomics and sentiment-based variables)

Notation

Details

Log yields

$$y_t^{(n)} \equiv -\frac{1}{n} p_t^{(n)}$$

where, $y_t^{(n)}$ denote the log yield of a n-maturity bond at time t
$p_t^{(n)}$ denote the natural logarithm price of this bond

- **holding period returns**

$$r_{t+\Delta}^{(n)} \equiv p_{t+\Delta}^{(n-\Delta)} - p_t^{(n)}$$

- **Excess Returns**

$$rx_{t+h/12}^{(n)} \equiv \text{holding period return } r_{t+h/12}^{(n)} - \text{1-period yield}$$

Spanning Hypothesis

- **SH is a central issue in macro-finance models** (Gürkaynak et al., 2007; Duffee, 2013; Bauer and Hamilton, 2018) EH

Spanning Hypothesis

- All relevant information to forecast yields and excess returns can be found on the term-structure.
- The yields curve fully spans all necessary information, and thus, no other variable already present in the term-structure should be necessary.
- It does not rules out the importance of macro variables (current or future).
- **Yield curve completely reflects and spans this information.**
- Influential works/factors:
  - ↳ Spanning: **Fama and Bliss (1987)** FB Details and **Cochrane and Piazzesi (2005)** CP Details
  - ↳ Not spanning: **Ludvigson and Ng (2009)** LN Details

A Deep-Learning Structure for Bond Premia

Partition of $\boldsymbol{Z}_t$

Proposition 1. *The state vector $\boldsymbol{Z}_t$ that encompasses all risks in the economy can be partitioned as $\boldsymbol{Z}_t = \left\{\boldsymbol{Z}_t^{y}, \boldsymbol{Z}_t^{y^{\complement}}\right\}$, in such a way that $\boldsymbol{Z}_t^{y}$ contains information solely from the yield curve, and $\boldsymbol{Z}_t^{y^{\complement}}$ any other information not found in the term structure.*

$\boldsymbol{Z}_t^{y}$ contains only yield curve variables [yields, forward rates]
$\boldsymbol{Z}_t^{y^{\complement}}$ contains any other variable (complement) [e.g., macro and sentiment-based variables]

We can summarize previous approaches with the following predictive regression:

$$rx_{t+h/12}^{(n)} = \beta^{\top} \boldsymbol{Z}_t + \epsilon_{t+h/12} \tag{1}$$

- **Spanning hypothesis** $\Rightarrow \boldsymbol{Z}_t = \{\boldsymbol{Z}_t^{y}\}$ (only yield curve information).
- Evidence against the **spanning hypothesis** $\Rightarrow \boldsymbol{Z}_t^{y^{\complement}} \neq \emptyset$.

Artificial Neural Network: A Primer

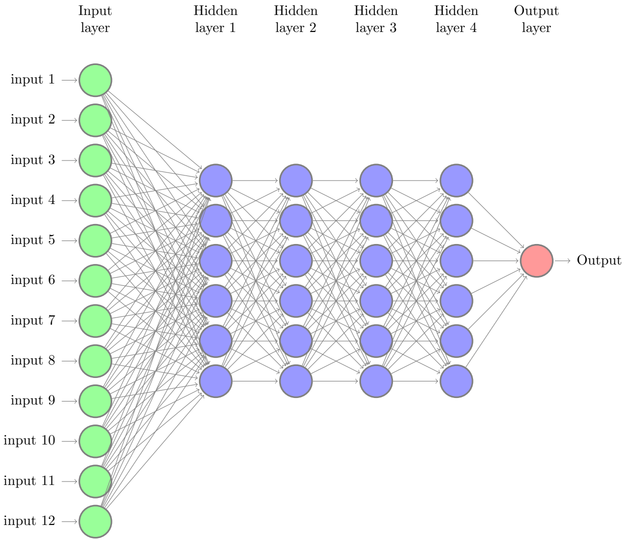
Idea: Attempt to replicate the brain architecture

↳ Many levels of processing information

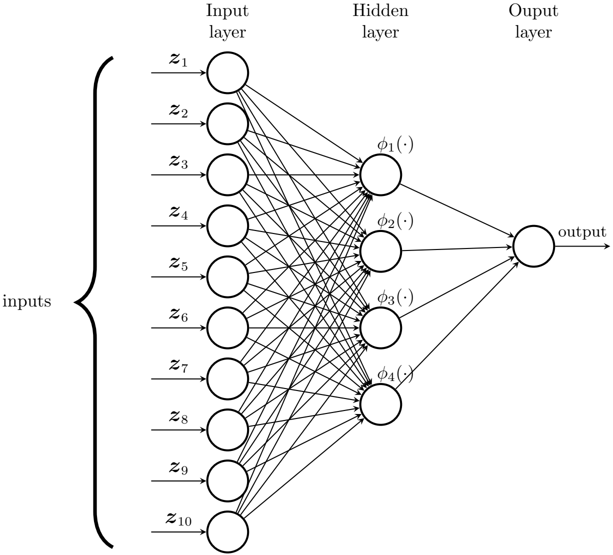
Goal: Extract complex non-linear combinations of the input

↳ Supervised Learning

↳ Conditioning on target (here, $rx^{(n)}_{t+h/12}$) and the inputs (here, $\boldsymbol{Z}_t$)



Artificial Neural Network: A Primer



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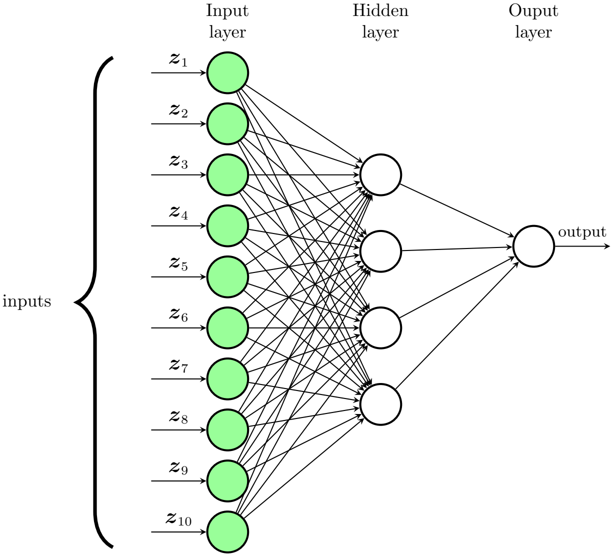

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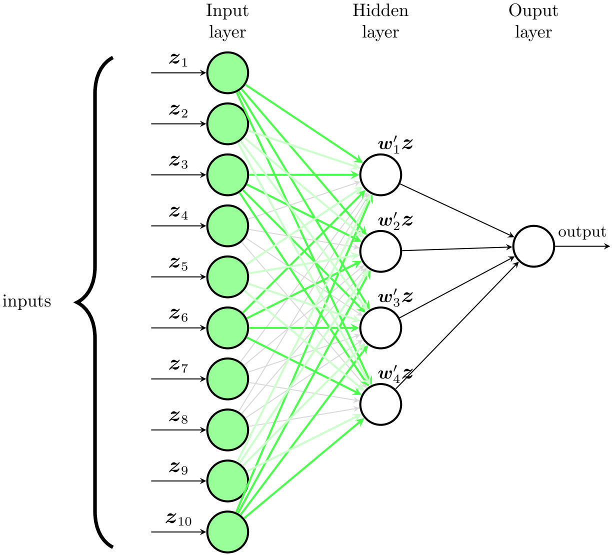

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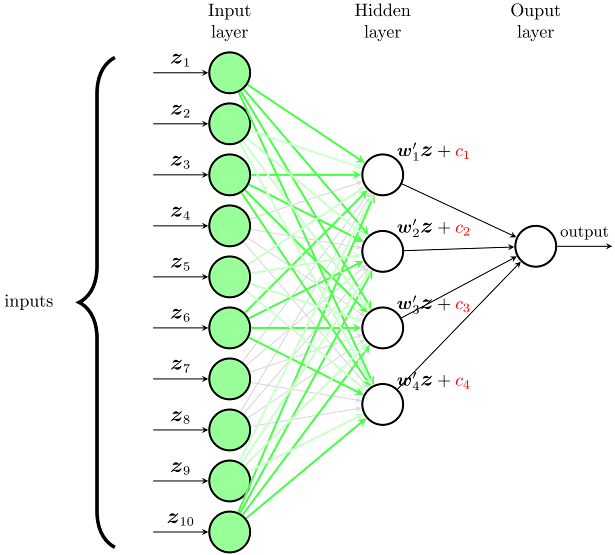

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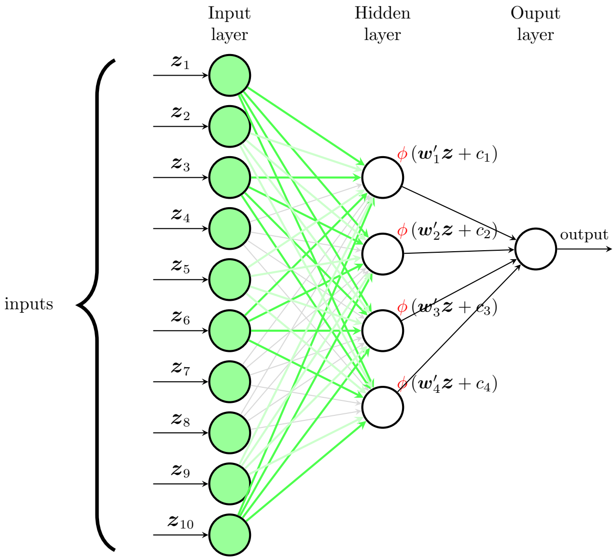

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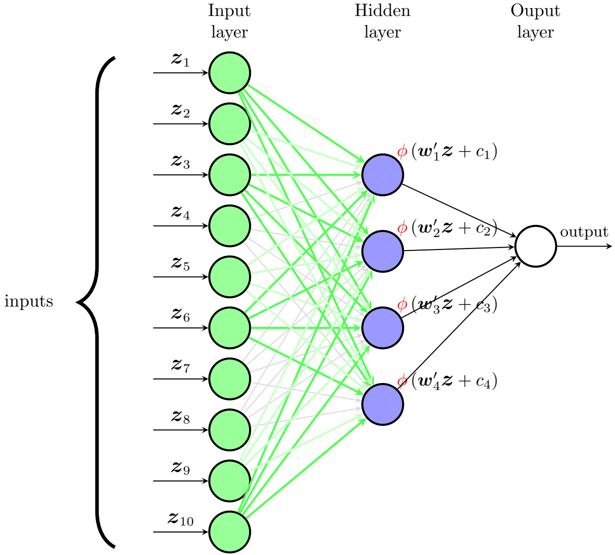

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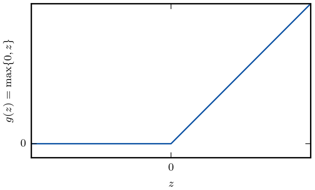

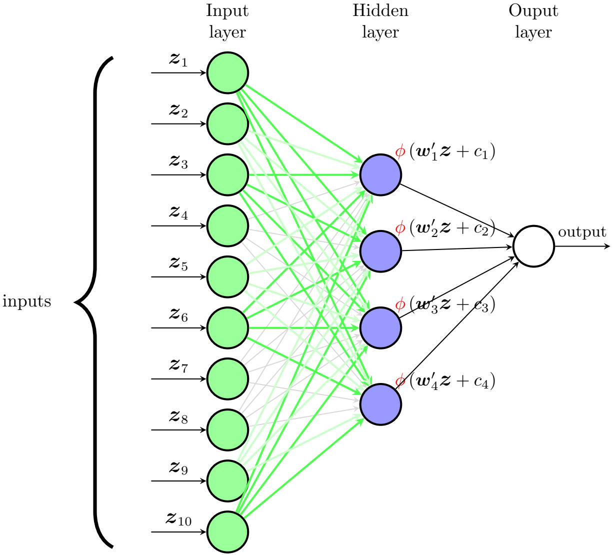

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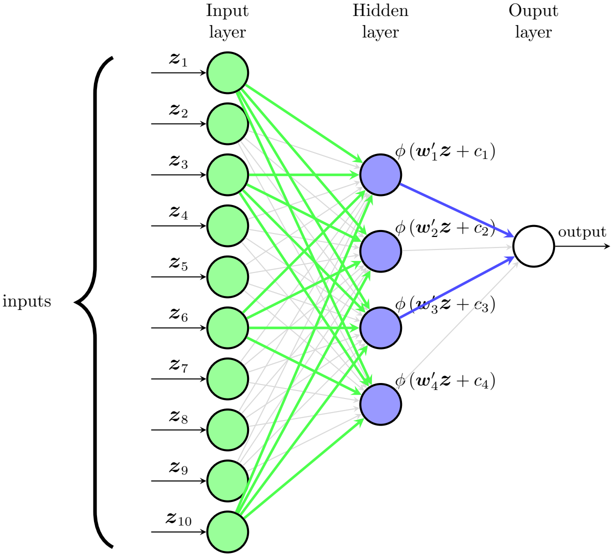

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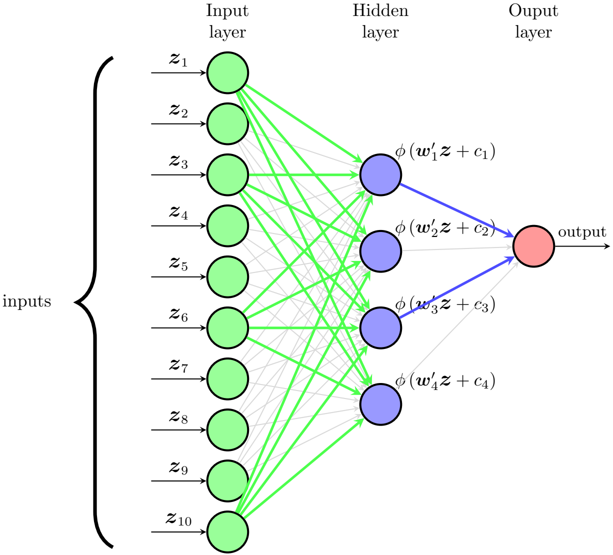


A Deep-Learning Structure for Bond Premia

- DNN defines a mapping such as $rx^{(n)}_{t+h/12} = g\left(\boldsymbol{Z}_t, \boldsymbol{\theta}_t\right)$ to *learn* the parameter $\boldsymbol{\theta}_t$ that provides the best function approximation.

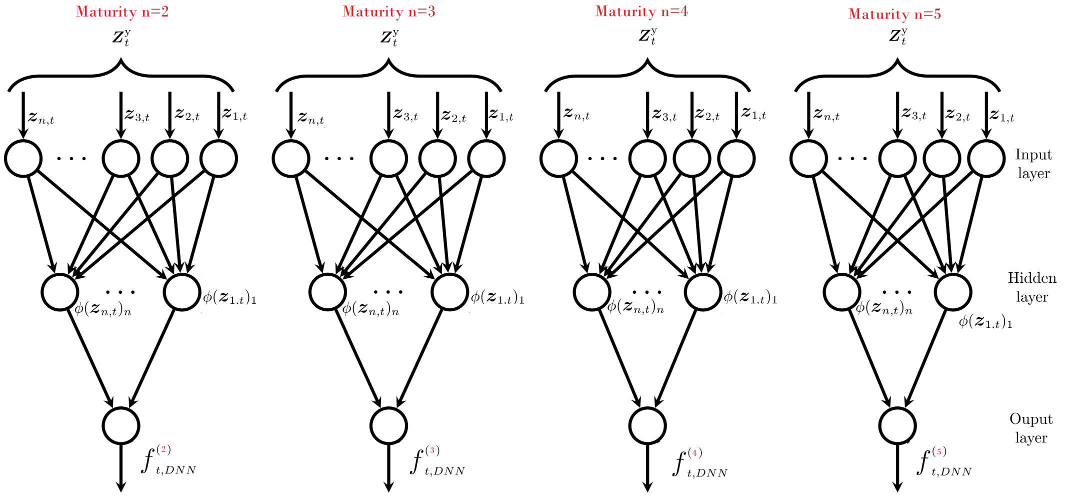
- Represented in a direct acyclic graph with a chain of functions $g\left(\boldsymbol{Z}_t\right) = g^{(L)}\left(\ldots\left(g^{(2)}\left(g^{(1)}\left(\boldsymbol{Z}_t\right)\right)\right)\right)$.

Universal Approximation Theorem (Hornik et al., 1989; Cybenko, 1989)

- Feedforward network with a linear output layer and **at least one hidden layer** with any activation function can approximate **any function**[1] from one finite-dimensional space to another with any desired nonzero amount of error.

  ↳ **Implication:** there exists a network large enough to achieve any degree of accuracy.

A Deep-Learning Structure for Bond Premia



DNN Factors

$$\begin{aligned}\frac{1}{4}\sum_{n=2}^{5} rx_{t+h/12}^{(n)} &= \tau_0 + \tau_1 \mathfrak{f}_{t,DNN}^{(2),h} + \tau_2 \mathfrak{f}_{t,DNN}^{(3),h} + \tau_3 \mathfrak{f}_{t,DNN}^{(4),h} + \tau_4 \mathfrak{f}_{t,DNN}^{(5),h} + \bar{\epsilon}_{t+h/12} \\ &= \boldsymbol{\tau}^{\top} \widehat{\mathfrak{F}}_t^{h} + \bar{\epsilon}_{t+h/12}\end{aligned} \tag{2}$$

where $\widehat{\mathfrak{F}}_t$ and $\boldsymbol{\tau}$ are 5×1 vectors given by $\widehat{\mathfrak{F}}_t \equiv \begin{bmatrix} 1 & \mathfrak{f}_{t,DNN}^{(2),h} & \mathfrak{f}_{t,DNN}^{(3),h} & \mathfrak{f}_{t,DNN}^{(4),h} & \mathfrak{f}_{t,DNN}^{(5),h} \end{bmatrix}^{\top}$, and $\boldsymbol{\tau} \equiv \begin{bmatrix} \tau_0 & \tau_1 & \tau_2 & \tau_3 & \tau_4 \end{bmatrix}^{\top}$.

- We **recursively** orthogonalize the excess returns generated by the deep neural network factor $\mathfrak{f}_{t,DNN}^{(n)}$, and denote it by $\boldsymbol{\xi}_t^{(n),h}$.

- The factor $\xi_{t+h/12}^{(n),h}$ that lies in an orthogonal vector to the space spanned by $\mathfrak{f}_{t,DNN}^{(n)}$, can be seen as all the information not spanned by the term-structure captured by $\mathfrak{f}_{t,DNN}^{(n)}$.

A Deep-Learning Structure for Bond Premia

Linear Rotation of the State Space

Proposition 2. *As in the dynamic term structure model of Joslin et al. (2014), $f(\xi^h_{t+h/12})$ complete and fill the unspanned factor in the state space, in a such a way that* $\left[\left(\boldsymbol{\tau}^\top \widehat{\mathfrak{F}}_t\right)^h_t, f(\xi^h_{t+h/12})\right]$ *and* $\boldsymbol{Z}_t$ *represent linear rotations of the same economy-wide risks underlying all tradable assets available to agents in the economy.*

A Deep-Learning Structure for Bond Premia

Linear Rotation of the State Space

Proposition 2. *As in the dynamic term structure model of Joslin et al. (2014), $f(\xi^h_{t+h/12})$ complete and fill the unspanned factor in the state space, in a such a way that [**Spanning Factor**, **Unspanning Factor**] and $\boldsymbol{Z}_t$ represent linear rotations of the same economy-wide risks underlying all tradable assets available to agents in the economy.*

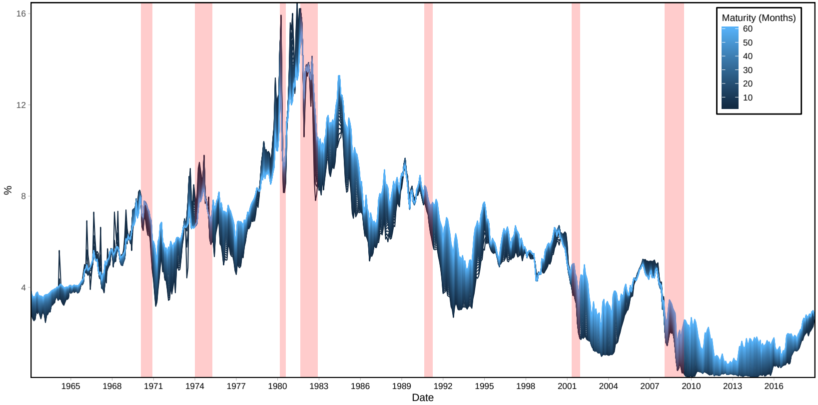
- Analogous to Joslin et al. (2014), we argue

  ↳ that the unspanned information in $\widehat{\xi}^h_{t+h/12}$ could be capturing macroeconomic information or sentiment measures not spanned by the term-structure.

Details

Data & Strategy

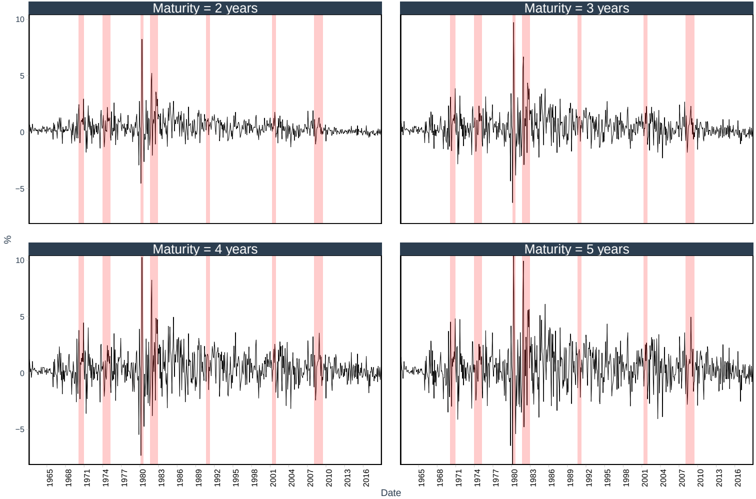
Derived zero-coupon bonds log yields for maturities (n) up to 60 months



- Full period: 1962:01 to 2017:12
- Period of evaluation: 1993:01 to 2017:12

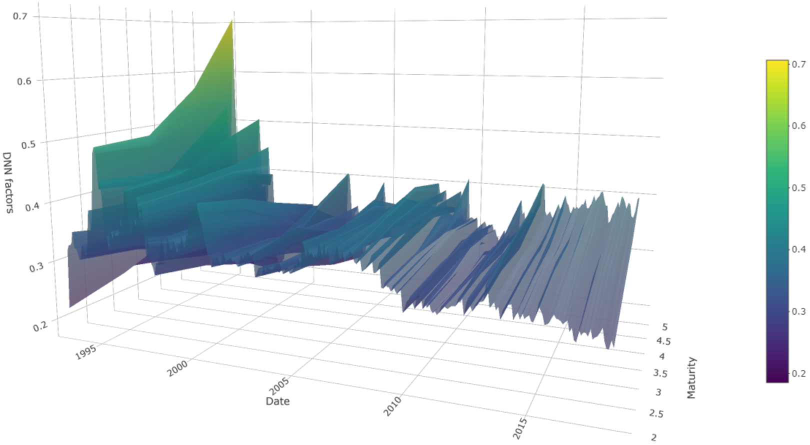
Treasuries Excess Returns

1-Month Bonds Excess Returns (1962-2017)



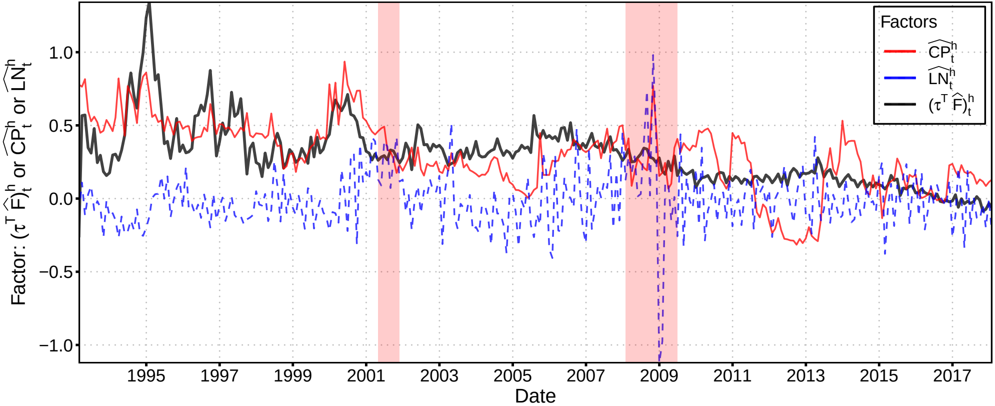
Empirical Results

Derived Factors $\mathfrak{f}_{t,DNN}^{(n),h}$ for **DNN 2** Generated Using the Set of Yields



Comparison with Other Factors from the Literature

Figure 1: Time Series of our Derived Factor $\left(\boldsymbol{\tau}^{\top}\widehat{\mathfrak{F}}_t\right)_t^h$, along with $\widehat{CP}_t^h$ and $\widehat{LN}_t^h$

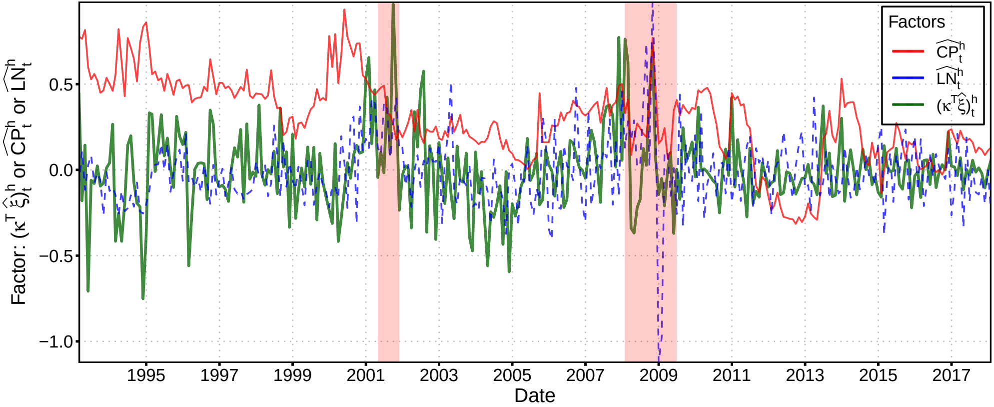


Empirical Results

Comparison with Other Factors from the Literature

Correlation

Figure 2: Time Series of our Derived Factor $\left(\kappa^\top \widehat{\xi}\right)_t^h$, along with $\widehat{CP}_t^h$ and $\widehat{LN}_t^h$



Empirical Results - Predictive Regressions Using $\left(\boldsymbol{\tau}^\top \widehat{\mathfrak{F}}_t\right)_t^h$ and $\left(\boldsymbol{\kappa}^\top \widehat{\boldsymbol{\xi}}\right)_t^{(-n),h}$ as State Variables

Details

	$rx^{(2)}_{t+h/12}$		$rx^{(3)}_{t+h/12}$		$rx^{(4)}_{t+h/12}$		$rx^{(5)}_{t+h/12}$	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$(\boldsymbol{\tau}^\top \widehat{\mathfrak{F}})_t^h$	0.811^{***}	0.811^{***}	0.943^{***}	0.943^{***}	1.065^{***}	1.065^{***}	1.181^{***}	1.181^{***}
	(0.131)	(0.119)	(0.199)	(0.188)	(0.264)	(0.253)	(0.325)	(0.312)
$\boldsymbol{M}_{\boldsymbol{\tau}^\top \widehat{\mathfrak{F}}}(\boldsymbol{\kappa}^\top \widehat{\boldsymbol{\xi}})_t^{(-n),h}$		0.779^{***}		0.789^{***}		0.807^{***}		0.848^{***}
		(0.180)		(0.219)		(0.288)		(0.318)
Constant	−0.010	−0.010	−0.003	−0.003	0.004	0.004	0.010	0.010
	(0.039)	(0.035)	(0.063)	(0.060)	(0.088)	(0.086)	(0.114)	(0.111)
Observations	300	300	300	300	300	300	300	300
Adjusted R^2	0.119	0.178	0.063	0.100	0.042	0.069	0.032	0.060

Note: $^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$

Empirical Results - Predictive Regressions with $\left(\boldsymbol{\tau}^\top \widehat{\boldsymbol{\mathfrak{F}}}_t\right)_t^h$ and $\left(\boldsymbol{\kappa}^\top \widehat{\boldsymbol{\xi}}\right)_t^{(-n),h}$, along with the Cochrane-Piazzesi and Ludvingson-Ng factors, and Fama-Bliss Regressions with Forward Spreads

Details

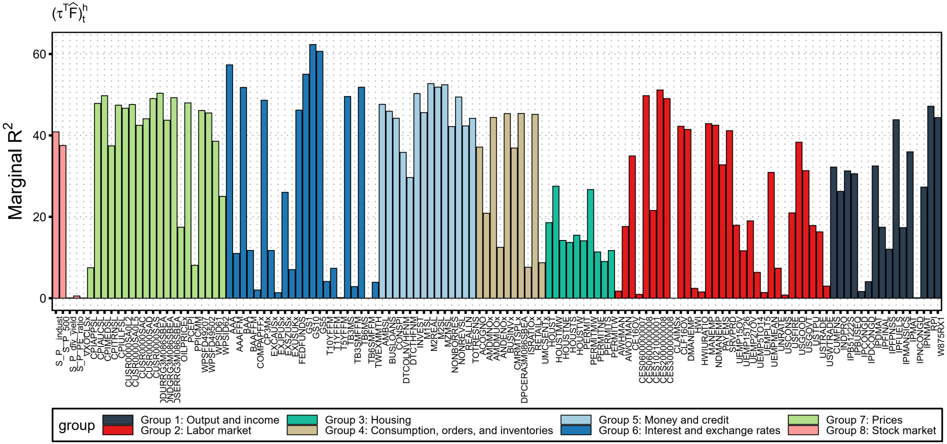
Panel A: $rx_{t+h/12}^{(2)}$

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$(\boldsymbol{\tau}^\top \widehat{\boldsymbol{\mathfrak{F}}})_t^h$	0.847***	0.842***	0.853***	0.824***	0.525***	0.582***	0.582***	0.614***
	(0.124)	(0.115)	(0.128)	(0.117)	(0.154)	(0.140)	(0.145)	(0.135)
$\boldsymbol{M}_{\boldsymbol{\tau}^\top \widehat{\boldsymbol{\mathfrak{F}}}}(\boldsymbol{\kappa}^\top \widehat{\boldsymbol{\xi}})_t^{(-2),h}$		0.658***		0.745***		0.704***		0.558***
		(0.172)		(0.182)		(0.182)		(0.185)
$\widehat{LN}_t^h$	0.617***	0.529***					0.559***	0.518***
	(0.127)	(0.120)					(0.110)	(0.110)
$fs_t^{(n,h)}$			−0.746	−0.225			−0.570	−0.172
			(0.476)	(0.438)			(0.437)	(0.429)
$\widehat{CP}_t^h$					0.454***	0.364***	0.465***	0.375***
					(0.126)	(0.112)	(0.112)	(0.109)
Constant	−0.013	−0.012	0.031	0.002	−0.060	−0.050	−0.031	−0.044
	(0.037)	(0.034)	(0.051)	(0.047)	(0.039)	(0.036)	(0.045)	(0.043)
Observations	300	300	300	300	300	300	300	300
Adjusted R^2	0.183	0.223	0.128	0.177	0.150	0.197	0.215	0.240

Note: $^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$

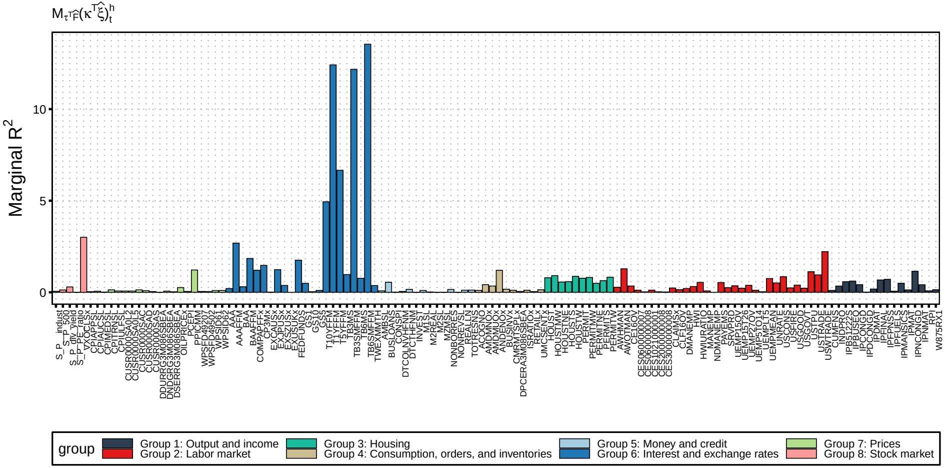
Empirical Results - Economic Interpretation

Marginal R^2 of the factor $\left(\boldsymbol{\tau}^\top \widehat{\mathfrak{F}}_t\right)_t^h$



Empirical Results - Economic Interpretation

Marginal R^2 of the factor $\boldsymbol{M}_{\boldsymbol{\tau}^\top \widehat{\mathfrak{F}}}(\boldsymbol{\kappa}^\top \widehat{\boldsymbol{\xi}})^h_{t+h/12}$



Out-of-Sample Forecasting Performance

- Set the out-of-sample period to range from 1997 : 01 to 2017 : 12, where the data from 1993 : 01 to 1996 : 12 is used to initiate the analysis.

- At each $\tau \in \tau_{OoS}$, we use all the previous information up to $\tau - 1$ to obtain the point forecast of $rx^{(n)}$ for the month τ.

Out-of-Sample R^2 (Campbell and Thompson, 2007; Gargano et al., 2019)

The out-of-sample R^2 is computed as

$$R^{2(n)}_{OoS,i} = 1 - \frac{\sum_{\tau \in \tau_{OoS}} \left(rx^{(n)}_{t+h/12|t} - \widehat{rx}^{(n)}_{t+h/12|t} \right)^2}{\sum_{\tau \in \tau_{OoS}} \left(rx^{(n)}_{t+h/12|t} - \overline{rx}^{(n)}_{t+h/12|t} \right)^2} \tag{3}$$

Out-of-Sample Forecasting Performance (R^2)

Additional Results

Regression	Maturity $n = 2$	Maturity $n = 3$	Maturity $n = 4$	Maturity $n = 5$
$rx_{t+h/12}^{(n)} = \beta_0 + \beta_1 (\boldsymbol{\tau}^\top \widehat{\boldsymbol{\mathfrak{F}}}_t)_t^h + \epsilon_{t+h/12}$	0.17	0.03	-0.02	-0.04
$rx_{t+h/12}^{(n)} = \beta_0 + \beta_1 \boldsymbol{M}_{\boldsymbol{\tau}^\top \widehat{\boldsymbol{\mathfrak{F}}}} (\boldsymbol{\kappa}^\top \widehat{\boldsymbol{\xi}})_t^h + \epsilon_{t+h/12}$	0.22	0.05	-0.01	-0.03
$rx_{t+h/12}^{(n)} = \beta_0 + \beta_1 \widehat{LN}_t^h + \epsilon_{t+h/12}$	0.12	-0.02	-0.06	-0.07
$rx_{t+h/12}^{(n)} = \beta_0 + \beta_1 fs_t^{(n,h)} + \epsilon_{t+h/12}$	0.18	0.05	0.00	-0.01
$rx_{t+h/12}^{(n)} = \beta_0 + \beta_1 \widehat{CP}_t^h + \epsilon_{t+h/12}$	0.15	-0.02	-0.08	-0.10

Out-of-Sample Forecasting Performance (R^2)

Additional Results

Regression	Maturity $n=2$	Maturity $n=3$	Maturity $n=4$	Maturity $n=5$
$rx_{t+h/12}^{(n)} = \beta_0 + \beta_1(\boldsymbol{\tau}^\top \widehat{\mathfrak{F}})_t^h + \beta_2 \widehat{LN}_t^h + \epsilon_{t+h/12}$	0.21	0.04	-0.03	-0.05
$rx_{t+h/12}^{(n)} = \beta_0 + \beta_1(\boldsymbol{\tau}^\top \widehat{\mathfrak{F}})_t^h + \beta_2 \boldsymbol{M}_{\boldsymbol{\tau}^\top \widehat{\mathfrak{F}}}(\boldsymbol{\kappa}^\top \widehat{\boldsymbol{\xi}})_t^{(-n),h} + \beta_3 \widehat{LN}_t^h + \epsilon_{t+h/12}$	0.23	0.04	-0.02	-0.05
$rx_{t+h/12}^{(n)} = \beta_0 + \beta_1(\boldsymbol{\tau}^\top \widehat{\mathfrak{F}})_t^h + \beta_2 fs_t^{(n,h)} + \epsilon_{t+h/12}$	0.26	0.08	0.02	-0.00
$rx_{t+h/12}^{(n)} = \beta_0 + \beta_1(\boldsymbol{\tau}^\top \widehat{\mathfrak{F}})_t^h + \beta_2 \boldsymbol{M}_{\boldsymbol{\tau}^\top \widehat{\mathfrak{F}}}(\boldsymbol{\kappa}^\top \widehat{\boldsymbol{\xi}})_t^{(-n),h} + \beta_3 fs_t^{(n,h)} + \epsilon_{t+h/12}$	0.27	0.08	0.02	-0.00
$rx_{t+h/12}^{(n)} = \beta_0 + \beta_1(\boldsymbol{\tau}^\top \widehat{\mathfrak{F}})_t^h + \beta_2 \widehat{CP}_t^h + \epsilon_{t+h/12}$	0.20	0.01	-0.06	-0.09
$rx_{t+h/12}^{(n)} = \beta_0 + \beta_1(\boldsymbol{\tau}^\top \widehat{\mathfrak{F}})_t^h + \beta_2 \boldsymbol{M}_{\boldsymbol{\tau}^\top \widehat{\mathfrak{F}}}(\boldsymbol{\kappa}^\top \widehat{\boldsymbol{\xi}})_t^{(-n),h} + \beta_3 \widehat{CP}_t^h + \epsilon_{t+h/12}$	0.22	0.01	-0.06	-0.08
$rx_{t+h/12}^{(n)} = \beta_0 + \beta_1(\boldsymbol{\tau}^\top \widehat{\mathfrak{F}})_t^h + \beta_2 \widehat{LN}_t^h + \beta_3 fs_t^{(n,h)} + \beta_4 \widehat{CP}_t^h + \epsilon_{t+h/12}$	0.19	-0.03	-0.10	-0.13
$rx_{t+h/12}^{(n)} = \beta_0 + \beta_1(\boldsymbol{\tau}^\top \widehat{\mathfrak{F}})_t^h + \beta_2 \boldsymbol{M}_{\boldsymbol{\tau}^\top \widehat{\mathfrak{F}}}(\boldsymbol{\kappa}^\top \widehat{\boldsymbol{\xi}})_t^{(-n),h} + \beta_3 \widehat{LN}_t^h + \beta_4 fs_t^{(n,h)} + \beta_5 \widehat{CP}_t^h + \epsilon_{t+h/12}$	0.19	-0.04	-0.11	-0.13

Conclusion

- I proposed a novel approach for deriving a **single state factor** consistent with a dynamic term-structure with unspanned risks.
- Making use of **deep neural networks** to uncover relationships in the term-structure, I build **a single state factor** that provides a good approximation to the space that spans all the information from the term-structure.
- I also introduced a way to obtain **unspanned risks from the yield curve** that is used to complete the state space.
- I show that this parsimonious number of state variables have predictive power for excess returns of bonds over 1-month holding period.
- Additionally, I provide an **intuitive interpretation of derived factors**, and show what information from macroeconomic variables and sentiment-based measures they can capture.

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Notation

Return

- **holding period returns**

$$\begin{aligned} r_{t+\Delta}^{(n)} &\equiv p_{t+\Delta}^{(n-\Delta)} - p_t^{(n)} \\ r_{t+h/12}^{(n)} &\equiv p_{t+h/12}^{(n-h/12)} - p_t^{(n)} = ny_t^{(n)} - (n-h/12)y_{t+h/12}^{(n-h/12)} \end{aligned} \quad (4)$$

- **Excess Returns**

$$\begin{aligned} rx_{t+h/12}^{(n)} &\equiv \text{holding period return } r_{t+h/12}^{(n)} \quad - \quad \text{1-period yield} \\ &= ny_t^{(n)} - (n-h/12)y_{t+h/12}^{(n-h/12)} - (h/12)y_t^{(h/12)} \end{aligned} \quad (5)$$

- **Forward rates** at time t for loans between time $t+n-h/12$ and $t+n$ as

$$\begin{aligned} f_t^{(n)} &\equiv p_t^{(n-h/12)} - \quad p_t^{(n)} \\ &= ny_t^{(n)} - (n-h/12)y_t^{(n-h/12)} \end{aligned} \quad (6)$$

Expectation Hypothesis

Return

Risk Premium: difference between a long rate and the expected average of future short rates.

$$y_t^{(n)} \equiv \underbrace{\frac{1}{n}\mathbb{E}_t\left(y_t^{(1/12)} + y_{t+1/12}^{(1/12)} + \ldots + y_{t+n-1/12}^{(1/12)}\right)}_{\text{expectations component}} + \underbrace{\frac{1}{n}\mathbb{E}_t\left(rx_{t+1/12}^{(n)} + rx_{t+2/12}^{(n-1/12)} + rx_{t+3/12}^{(n-2/12)} + \ldots + rx_{t+n-1/12}^{(2/12)}\right)}_{\text{yield risk premium}} \tag{7}$$

Expectation Hypothesis

Return

Assuming that the agents' information set at time t can be summarized by a state vector $\mathbf{Z}_t$

$$y_t^{(n)} = \frac{1}{n}\left(\sum_{j=0}^{12\cdot n/h-1} \mathbb{E}\left[y_{t+j\cdot h/12}^{(h/12)} | \mathbf{Z}_t\right]\right) + \frac{1}{n}\left(\sum_{j=0}^{12\cdot n/h-1}\left[rx_{t+h/12(j+1)}^{(n-j\cdot h/12)} | \mathbf{Z}_t\right]\right) . \quad (8)$$

$\mathbf{Z}_t$ should contain all the information used by investors to forecast at time t the excess-returns for all future periods.

Fama and Bliss (1987)

Return

- Fama and Bliss (1987) builds forward rates spreads and use these variables as covariates.
- Forward rate spread between of a n-year maturity bond: $fs_t^{(n,h)} \equiv f_t^{(n)} - \quad y_t^{(h/12)}(h/12)$.

Predictive Regression

$$rx_{t+h/12}^{(n)} = \beta_0 + \beta_1 fs_t^{(n,h)} + \epsilon_{t+h/12} \quad . \tag{9}$$

Cochrane and Piazzesi (2005)

Return

- Cochrane and Piazzesi (2005) derive a single factor to use as predictor (CP_t^h).
- First, they estimate (CP_t^h) as

$$
\begin{aligned}
\frac{1}{4}\sum_{n=2}^{5} rx_{t+h/12}^{(n)} &= \gamma_0 + \gamma_1 f_t^{(1)} + \gamma_2 f_t^{(2)} + \gamma_3 f_t^{(3)} + \gamma_4 f_t^{(4)} + \gamma_5 f_t^{(5)} + \bar{\epsilon}_{t+h/12} \\
\overline{rx}_{t+h/12} &= \underbrace{\gamma^\top \mathbf{f}_t}_{CP_t^h} + \bar{\epsilon}_{t+h/12}
\end{aligned} \tag{10}
$$

where $\boldsymbol{f}$ and $\boldsymbol{\gamma}$ are 6×1 vectors given by $\boldsymbol{f} \equiv \begin{bmatrix} 1 & f_t^{(1)} & f_t^{(2)} & f_t^{(3)} & f_t^{(4)} & f_t^{(5)} \end{bmatrix}^\top$, and $\boldsymbol{\gamma} \equiv [\gamma_0 \quad \gamma_1 \quad \gamma_2 \quad \gamma_3 \quad \gamma_4 \quad \gamma_5]^\top$.

Predictive Regression

$$
rx_{t+h/12}^{(n)} = \beta_0 + \beta_1 \widehat{CP}_t^h + \epsilon_{t+h/12} \quad . \tag{11}
$$

Ludvigson and Ng (2009)

Return

- Ludvigson and Ng (2009) use a large panel of macro variables, and build a single linear combination (LN_t^h) out of the first i estimated principal components ($\hat{g}_{i,t}$).

- First, they estimate (LN_t^h) as

$$\begin{aligned} \frac{1}{4}\sum_{n=2}^{5} rx_{t+h/12}^{(n)} &= \lambda_0 + \lambda_1\hat{g}_{1,t} + \lambda_2\hat{g}_{1,t}^3 + \lambda_3\hat{g}_{3,t} + \lambda_4\hat{g}_{4,t} + \lambda_5\hat{g}_{8,t} + \bar{\epsilon}_{t+h/12} \\ \overline{rx}_{t+h/12} &= \underbrace{\boldsymbol{\lambda}^\top \widehat{\boldsymbol{G}}_t}_{LN_t^h} + \bar{\epsilon}_{t+h/12} \end{aligned} \tag{12}$$

where $\widehat{\boldsymbol{G}}_t$ and $\boldsymbol{\lambda}$ are 5×1 vectors given by $\widehat{\boldsymbol{G}}_t \equiv \begin{bmatrix}\hat{g}_{1,t} & \hat{g}_{1,t}^3 & \hat{g}_{3,t} & \hat{g}_{5,t} & \hat{g}_{8,t}\end{bmatrix}^\top$, and $\boldsymbol{\lambda} \equiv \begin{bmatrix}\lambda_1 & \lambda_2 & \lambda_3 & \lambda_4 & \lambda_5\end{bmatrix}^\top$.

Predictive Regression

$$rx_{t+h/12}^{(n)} = \beta_0 + \beta_1 \widehat{LN}_t^h + \epsilon_{t+h/12} \quad . \tag{13}$$

Algorithm 1: Recursively generated factors with updated parameters

Initialization: Start with a set of information from the term structure collected in $\boldsymbol{Z}^{y}$. Partitionate your sample $\left\{t_0, \ldots, t_{split}, \tau, \tau+1, \ldots, T\right\}$ between the data to be used to initialize the process $\left\{t_0, \ldots, t_{split}\right\}$, and to obtain the recursively generated factors $\{\tau, \tau+1, \ldots, T\}$;

for $n \in \{2,3,4,5\}$ **do**

  for $t \in \{\tau, \tau+1, \ldots, T\}$ **do**

    Feed DNN_i with lagged data $\boldsymbol{Z}^{y}_{t-1} = \left\{\boldsymbol{z}^{y}_{t_0}, \boldsymbol{z}^{y}_{t_0+1}, \ldots, \boldsymbol{z}^{y}_{t-1}\right\}$ to learn/aproximate with output $rx_t^{(n)}$, and use the last 10% of the data for validation;

    Obtain the learned parameters;

$$\widehat{\mathfrak{f}}^{(n),h}_{t,DNN} \leftarrow g\left(\boldsymbol{Z}^{y}_{t-1}, \boldsymbol{\theta}_{t-1}\right)$$

    Obtain the t-*th* element that lies in the orthogonal vector from the space generated by the $\mathfrak{f}^{(n),h}_{t-1,DNN}$ through:

$$\widehat{\xi}^{(n),h}_{t} \leftarrow rx_t^{(n)} - \widehat{\beta}_0 - \widehat{\beta}_1 \mathfrak{f}^{(n),h}_{t-1,DNN_i}$$

  end

end

Return

Algorithm 2: Recursively generated factors with updated parameters

Result:

$$\widehat{\mathfrak{F}}_{t,DNN_i} \equiv \begin{bmatrix} \widehat{\mathfrak{f}}^{(2),h}_{t,DNN_i} \\ \widehat{\mathfrak{f}}^{(3),h}_{t,DNN_i} \\ \widehat{\mathfrak{f}}^{(4),h}_{t,DNN_i} \\ \widehat{\mathfrak{f}}^{(5),h}_{t,DNN_i} \end{bmatrix} = \begin{bmatrix} \widehat{\mathfrak{f}}^{(2),h}_{\tau,DNN_i} & \widehat{\mathfrak{f}}^{(3),h}_{\tau,DNN_i} & \widehat{\mathfrak{f}}^{(4),h}_{\tau,DNN_i} & \widehat{\mathfrak{f}}^{(5),h}_{\tau,DNN_i} \\ \widehat{\mathfrak{f}}^{(2),h}_{\tau+1,DNN_i} & \widehat{\mathfrak{f}}^{(3),h}_{\tau+1,DNN_i} & \widehat{\mathfrak{f}}^{(4),h}_{\tau+1,DNN_i} & \widehat{\mathfrak{f}}^{(5),h}_{\tau+1,DNN_i} \\ \vdots & \vdots & \vdots & \vdots \\ \widehat{\mathfrak{f}}^{(2),h}_{T,DNN_i} & \widehat{\mathfrak{f}}^{(3),h}_{T,DNN_i} & \widehat{\mathfrak{f}}^{(4),h}_{T,DNN_i} & \widehat{\mathfrak{f}}^{(5),h}_{T,DNN_i} \end{bmatrix}$$

And,

$$\widehat{\xi}^h_t \equiv \begin{bmatrix} \widehat{\xi}^{(2),h}_{\tau,DNN_i} & \widehat{\xi}^{(3),h}_{\tau,DNN_i} & \widehat{\xi}^{(4),h}_{\tau,DNN_i} & \widehat{\xi}^{(5),h}_{\tau,DNN_i} \\ \widehat{\xi}^{(2),h}_{\tau+1,DNN_i} & \widehat{\xi}^{(3),h}_{\tau+1,DNN_i} & \widehat{\xi}^{(4),h}_{\tau+1,DNN_i} & \widehat{\xi}^{(5),h}_{\tau+1,DNN_i} \\ \vdots & \vdots & \vdots & \vdots \\ \widehat{\xi}^{(2),h}_{T,DNN_i} & \widehat{\xi}^{(3),h}_{T,DNN_i} & \widehat{\xi}^{(4),h}_{T,DNN_i} & \widehat{\xi}^{(5),h}_{T,DNN_i} \end{bmatrix}$$

An Illustrative Term-Structure Model

Return

The no-arbitrage assumption rely on the fundamental asset pricing equation:

$$P_t^{(n)} = \mathbb{E}_t \left(\mathcal{M}_{t+1} P_{t+1}^{(n-1)} \right) \tag{14}$$

where

- $P_t^{(n)}$ is the price of a bond,

- $\mathcal{M}_{t+h/12}$ is the stochastic discount factor (SDF).

SDF:

$$\mathcal{M}_{t+h/12} = \exp^{-r_t \frac{1}{2} \Lambda_t^\top \Lambda_t - \Lambda_t^\top \epsilon_{t+h/12}} \tag{15}$$

where Λ_t is the market prices of the risks, i.e., the amount of compensation required by investors to face the unit normal shock $\epsilon_{t+h/12}$.

$$r_t = \rho_0 + \rho_1 \boldsymbol{Z_t} \quad . \tag{16}$$

An Illustrative Term-Structure Model

Return

- Define $\boldsymbol{Z}_t = \left\{ \boldsymbol{Z}_t^{\mathsf{y}}, \boldsymbol{Z}_t^{\mathsf{y}^{\complement}} \right\}$

- Dynamics of $\boldsymbol{Z}_t$ that capture all the risks of the economy following a Gaussian VAR process given by:

$$\begin{aligned} \begin{bmatrix} \boldsymbol{Z}_t^{\mathsf{y}} \\ \boldsymbol{Z}_t^{\mathsf{y}^{\complement}} \end{bmatrix} &= \boldsymbol{\mu} + \boldsymbol{\Phi} \begin{bmatrix} \boldsymbol{Z}_{t-1}^{\mathsf{y}} \\ \boldsymbol{Z}_{t-1}^{\mathsf{y}^{\complement}} \end{bmatrix} + \boldsymbol{\Sigma}\epsilon_t \\ \boldsymbol{Z}_t &= \boldsymbol{\mu} + \boldsymbol{\Phi}\boldsymbol{Z}_{t-1} + \boldsymbol{\Sigma}\epsilon_t \qquad \epsilon_t \sim N(0, \boldsymbol{I}) \end{aligned} \tag{17}$$

where $\boldsymbol{\mu}$ is a a $k \times 1$ vector, and $\boldsymbol{\Phi}$ and $\boldsymbol{\Sigma}$ are $k \times k$ matrices, being k the number of state variables.

An Illustrative Term-Structure Model

Details

- In a similar fashion to Joslin et al. (2014), we can write:

$$\boldsymbol{Z}_t^{y^{\complement}} = \gamma_0 + \gamma_1 \boldsymbol{Z}_t^{y} + \boldsymbol{M}_{\boldsymbol{Z}_t^{y}} \boldsymbol{Z}_t^{y^{\complement}} \tag{18}$$

where $\boldsymbol{M}_{\boldsymbol{Z}_t^{y}} \boldsymbol{Z}_t^{y^{\complement}}$ is the annihilator matrix of the space spanned by $\boldsymbol{Z}_t^{y}$, i.e.,

$$\boldsymbol{M}_{\boldsymbol{Z}_t^{y}} \boldsymbol{Z}_t^{y^{\complement}} \equiv \boldsymbol{Z}_t^{y^{\complement}} - \text{Proj}\left[\boldsymbol{Z}_t^{y^{\complement}} | \boldsymbol{Z}_t^{y}\right] \tag{19}$$

In our methodology,

- $\boldsymbol{Z}_t^{y}$ is given by the derived factor $\left(\boldsymbol{\tau}^\top \widehat{\mathfrak{F}}_t\right)_t^h$
- $\boldsymbol{Z}_t^{y^{\complement}}$ by a function of $\xi_{t+h/12}^h$ as $f(\xi_{t+h/12}^h)$

Correlation Matrix

Return

	$(\tau^\top \widehat{\mathfrak{F}})_t^h$	$M_{\tau^\top \widehat{\mathfrak{F}}}(\kappa^\top \widehat{\xi})_t^h$	$M_{\tau^\top \widehat{\mathfrak{F}}}(\kappa^\top \widehat{\xi})_t^{(-2),h}$	$M_{\tau^\top \widehat{\mathfrak{F}}}(\kappa^\top \widehat{\xi})_t^{(-3),h}$	$M_{\tau^\top \widehat{\mathfrak{F}}}(\kappa^\top \widehat{\xi})_t^{(-4),h}$	$M_{\tau^\top \widehat{\mathfrak{F}}}(\kappa^\top \widehat{\xi})_t^{(-5),h}$	$\widehat{CP}_t^h$	$\widehat{LN}_t^h$
$(\tau^\top \widehat{\mathfrak{F}})_t^h$	1	0	0	0	0	0	0.556	-0.059
$M_{\tau^\top \widehat{\mathfrak{F}}}(\kappa^\top \widehat{\xi})_t^h$	0	1	0.995	0.912	0.904	0.919	0.129	0.171
$M_{\tau^\top \widehat{\mathfrak{F}}}(\kappa^\top \widehat{\xi})_t^{(-2),h}$	0	0.995	1	0.938	0.900	0.888	0.135	0.174
$M_{\tau^\top \widehat{\mathfrak{F}}}(\kappa^\top \widehat{\xi})_t^{(-3),h}$	0	0.912	0.938	1	0.947	0.849	0.170	0.203
$M_{\tau^\top \widehat{\mathfrak{F}}}(\kappa^\top \widehat{\xi})_t^{(-4),h}$	0	0.904	0.900	0.947	1	0.959	0.173	0.204
$M_{\tau^\top \widehat{\mathfrak{F}}}(\kappa^\top \widehat{\xi})_t^{(-5),h}$	0	0.919	0.888	0.849	0.959	1	0.146	0.178
$\widehat{CP}_t^h$	0.556	0.129	0.135	0.170	0.173	0.146	1	-0.007
$\widehat{LN}_t^h$	-0.059	0.171	0.174	0.203	0.204	0.178	-0.007	1

Empirical Results - Predictive Regressions Using $\left(\boldsymbol{\tau}^{\top}\widehat{\mathfrak{F}}_t\right)^h_t$, $\left(\boldsymbol{\kappa}^{\top}\widehat{\boldsymbol{\xi}}\right)^h_t$ and $\left(\boldsymbol{\kappa}^{\top}\widehat{\boldsymbol{\xi}}\right)^{(-n),h}_t$ as State Variables

Return

Panel A: $rx^{(2)}_{t+h/12}$

	DNN 1			DNN 2			DNN 3		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
$(\boldsymbol{\tau}^{\top}\widehat{\mathfrak{F}})^h_t$	0.810***	0.810***	0.810***	0.811***	0.811***	0.811***	1.419***	1.419***	1.419***
	(0.160)	(0.149)	(0.147)	(0.131)	(0.119)	(0.119)	(0.414)	(0.377)	(0.356)
$\boldsymbol{M}_{\boldsymbol{\tau}^{\top}\widehat{\mathfrak{F}}}(\boldsymbol{\kappa}^{\top}\widehat{\boldsymbol{\xi}})^{(-2),h}_t$		0.760***			0.779***			0.875***	
		(0.204)			(0.180)			(0.211)	
$\boldsymbol{M}_{\boldsymbol{\tau}^{\top}\widehat{\mathfrak{F}}}(\boldsymbol{\kappa}^{\top}\widehat{\boldsymbol{\xi}})^{h}_t$			0.591***			0.525***			0.679***
			(0.139)			(0.126)			(0.138)
Constant	−0.010	−0.010	−0.010	−0.010	−0.010	−0.010	−0.189*	−0.189*	−0.189**
	(0.054)	(0.050)	(0.049)	(0.039)	(0.035)	(0.035)	(0.110)	(0.101)	(0.094)
Observations	300	300	300	300	300	300	300	300	300
Adjusted R^2	0.100	0.148	0.159	0.119	0.178	0.175	0.046	0.105	0.124

Note: *p<0.1; **p<0.05; ***p<0.01

Empirical Results - Predictive Regressions Using $\left(\boldsymbol{\tau}^\top \widehat{\mathfrak{F}}_t\right)_t^h$, $\left(\boldsymbol{\kappa}^\top \widehat{\boldsymbol{\xi}}\right)_t^h$ and $\left(\boldsymbol{\kappa}^\top \widehat{\boldsymbol{\xi}}\right)_t^{(-n),h}$ as State Variables

Return

Panel B: $rx_{t+h/12}^{(3)}$

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
$(\boldsymbol{\tau}^\top \widehat{\mathfrak{F}})_t^h$	0.959***	0.959***	0.959***	0.943***	0.943***	0.943***	1.175*	1.175**	1.175**
	(0.248)	(0.234)	(0.233)	(0.199)	(0.188)	(0.184)	(0.630)	(0.566)	(0.559)
$\boldsymbol{M}_{\boldsymbol{\tau}^\top \widehat{\mathfrak{F}}}(\boldsymbol{\kappa}^\top \widehat{\boldsymbol{\xi}})_{t+h/12}^{(-3),h}$		0.799***			0.789***			0.984***	
		(0.234)			(0.219)			(0.236)	
$\boldsymbol{M}_{\boldsymbol{\tau}^\top \widehat{\mathfrak{F}}}(\boldsymbol{\kappa}^\top \widehat{\boldsymbol{\xi}})_{t+h/12}^{h}$			0.765***			0.757***			0.929***
			(0.225)			(0.205)			(0.224)
Constant	−0.008	−0.008	−0.008	−0.003	−0.003	−0.003	−0.072	−0.072	−0.072
	(0.087)	(0.082)	(0.082)	(0.063)	(0.060)	(0.059)	(0.169)	(0.153)	(0.150)
Observations	300	300	300	300	300	300	300	300	300
Adjusted R^2	0.055	0.092	0.093	0.063	0.100	0.109	0.010	0.067	0.067

Note: $^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$

Empirical Results - Predictive Regressions Using $\left(\boldsymbol{\tau}^\top \widehat{\mathfrak{F}}_t\right)_t^h$, $\left(\boldsymbol{\kappa}^\top \widehat{\boldsymbol{\xi}}\right)_t^h$ and $\left(\boldsymbol{\kappa}^\top \widehat{\boldsymbol{\xi}}\right)_t^{(-n),h}$ as State Variables

Return

Panel C: $rx_{t+h/12}^{(4)}$

$(\boldsymbol{\tau}^\top \widehat{\mathfrak{F}})_t^h$	1.073***	1.073***	1.073***	1.065***	1.065***	1.065***	0.864	0.864	0.864
	(0.334)	(0.320)	(0.317)	(0.264)	(0.253)	(0.248)	(0.835)	(0.759)	(0.755)
$\boldsymbol{M}_{\boldsymbol{\tau}^\top \widehat{\mathfrak{F}}}(\boldsymbol{\kappa}^\top \widehat{\boldsymbol{\xi}})_t^{(-4),h}$		0.795***			0.807***			1.038***	
		(0.291)			(0.288)			(0.289)	
$\boldsymbol{M}_{\boldsymbol{\tau}^\top \widehat{\mathfrak{F}}}(\boldsymbol{\kappa}^\top \widehat{\boldsymbol{\xi}})_t^h$			0.902***			0.945***			1.144***
			(0.312)			(0.284)			(0.313)
Constant	0.002	0.002	0.002	0.004	0.004	0.004	0.063	0.063	0.063
	(0.120)	(0.116)	(0.115)	(0.088)	(0.086)	(0.085)	(0.228)	(0.209)	(0.207)
Observations	300	300	300	300	300	300	300	300	300
Adjusted R^2	0.036	0.060	0.063	0.042	0.069	0.080	0.001	0.046	0.046

Note: *p<0.1; **p<0.05; ***p<0.01

Empirical Results - Predictive Regressions Using $\left(\boldsymbol{\tau}^\top \widehat{\mathfrak{F}}_t\right)_t^h$, $\left(\boldsymbol{\kappa}^\top \widehat{\boldsymbol{\xi}}\right)_t^h$ and $\left(\boldsymbol{\kappa}^\top \widehat{\boldsymbol{\xi}}\right)_t^{(-n),h}$ as State Variables

Return

Panel D: $rx_{t+h/12}^{(5)}$

$(\boldsymbol{\tau}^\top \widehat{\mathfrak{F}})_t^h$	1.158***	1.158***	1.158***	1.181***	1.181***	1.181***	0.542	0.542	0.542
	(0.415)	(0.395)	(0.398)	(0.325)	(0.312)	(0.309)	(1.025)	(0.949)	(0.939)
$\boldsymbol{M}_{\boldsymbol{\tau}^\top \widehat{\mathfrak{F}}}(\boldsymbol{\kappa}^\top \widehat{\boldsymbol{\xi}})_t^{(-5),h}$		0.854**			0.848***			1.069***	
		(0.336)			(0.318)			(0.339)	
$\boldsymbol{M}_{\boldsymbol{\tau}^\top \widehat{\mathfrak{F}}}(\boldsymbol{\kappa}^\top \widehat{\boldsymbol{\xi}})_t^h$			1.000**			1.081***			1.322***
			(0.398)			(0.363)			(0.404)
Constant	0.017	0.017	0.017	0.010	0.010	0.010	0.198	0.198	0.198
	(0.152)	(0.146)	(0.147)	(0.114)	(0.111)	(0.111)	(0.284)	(0.267)	(0.263)
Observations	300	300	300	300	300	300	300	300	300
Adjusted R^2	0.025	0.049	0.046	0.032	0.060	0.062	-0.002	0.033	0.036

Note: $^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$

Empirical Results - Predictive Regressions with $\left(\boldsymbol{\tau}^\top \widehat{\mathfrak{F}}_t\right)_t^h$ and $\left(\boldsymbol{\kappa}^\top \widehat{\boldsymbol{\xi}}\right)_t^{(-n),h}$, along with the Cochrane-Piazzesi and Ludvingson-Ng factors, and Fama-Bliss Regressions with Forward Spreads

Return

Panel B: $rx^{(3)}_{t+h/12}$

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$(\boldsymbol{\tau}^\top \widehat{\mathfrak{F}})_t^h$	0.996***	0.989***	0.940***	0.947***	0.559**	0.648***	0.626***	0.719***
	(0.190)	(0.184)	(0.199)	(0.188)	(0.245)	(0.234)	(0.238)	(0.237)
$\boldsymbol{M}_{\boldsymbol{\tau}^\top \widehat{\mathfrak{F}}}(\boldsymbol{\kappa}^\top \widehat{\boldsymbol{\xi}})_t^{(-3),h}$		0.620***		0.852***		0.692***		0.585**
		(0.209)		(0.228)		(0.226)		(0.237)
$\widehat{LN}_t^h$	0.921***	0.800***					0.900***	0.823***
	(0.209)	(0.201)					(0.194)	(0.191)
$fs_t^{(n,h)}$			−0.215	0.410			−0.053	0.394
			(0.554)	(0.532)			(0.525)	(0.542)
$\widehat{CP}_t^h$					0.608***	0.467**	0.583***	0.437**
					(0.205)	(0.195)	(0.188)	(0.198)
Constant	−0.007	−0.006	0.021	−0.049	−0.070	−0.054	−0.064	−0.098
	(0.060)	(0.059)	(0.091)	(0.087)	(0.063)	(0.061)	(0.082)	(0.082)
Observations	300	300	300	300	300	300	300	300
Adjusted R^2	0.120	0.141	0.060	0.099	0.084	0.111	0.136	0.151

Note: $^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$

Empirical Results - Predictive Regressions with $\left(\boldsymbol{\tau}^\top \widehat{\boldsymbol{\mathfrak{F}}}_t\right)_t^h$ and $\left(\boldsymbol{\kappa}^\top \widehat{\boldsymbol{\xi}}\right)_t^{(-n),h}$, along with the Cochrane-Piazzesi and Ludvingson-Ng factors, and Fama-Bliss Regressions with Forward Spreads

Return

Panel C: $rx^{(4)}_{t+h/12}$

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$(\boldsymbol{\tau}^\top \widehat{\boldsymbol{\mathfrak{F}}})_t^h$	1.135***	1.127***	1.082***	1.108***	0.547	0.651**	0.685**	0.790**
	(0.254)	(0.247)	(0.270)	(0.257)	(0.335)	(0.323)	(0.329)	(0.329)
$\boldsymbol{M}_{\boldsymbol{\tau}^\top \widehat{\boldsymbol{\mathfrak{F}}}}(\boldsymbol{\kappa}^\top \widehat{\boldsymbol{\xi}})_t^{(-4),h}$		0.609**		0.872***		0.688**		0.555**
		(0.262)		(0.289)		(0.291)		(0.274)
$\widehat{LN}_t^h$	1.218***	1.079***					1.222***	1.118***
	(0.307)	(0.287)					(0.285)	(0.273)
$fs_t^{(n,h)}$			0.260	0.665			0.386	0.655
			(0.622)	(0.595)			(0.593)	(0.587)
$\widehat{CP}_t^h$					0.822***	0.657**	0.755***	0.606**
					(0.290)	(0.276)	(0.265)	(0.272)
Constant	−0.0003	0.0002	−0.038	−0.103	−0.085	−0.068	−0.144	−0.171
	(0.085)	(0.084)	(0.130)	(0.124)	(0.089)	(0.087)	(0.121)	(0.118)
Observations	300	300	300	300	300	300	300	300
Adjusted R^2	0.095	0.108	0.039	0.070	0.063	0.081	0.112	0.122

Note: $^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$

Empirical Results - Predictive Regressions with $\left(\boldsymbol{\tau}^{\top}\widehat{\mathfrak{F}}_t\right)_t^h$ and $\left(\boldsymbol{\kappa}^{\top}\widehat{\boldsymbol{\xi}}\right)_t^{(-n),h}$, along with the Cochrane-Piazzesi and Ludvingson-Ng factors, and Fama-Bliss Regressions with Forward Spreads

Return

Panel D: $rx^{(5)}_{t+h/12}$

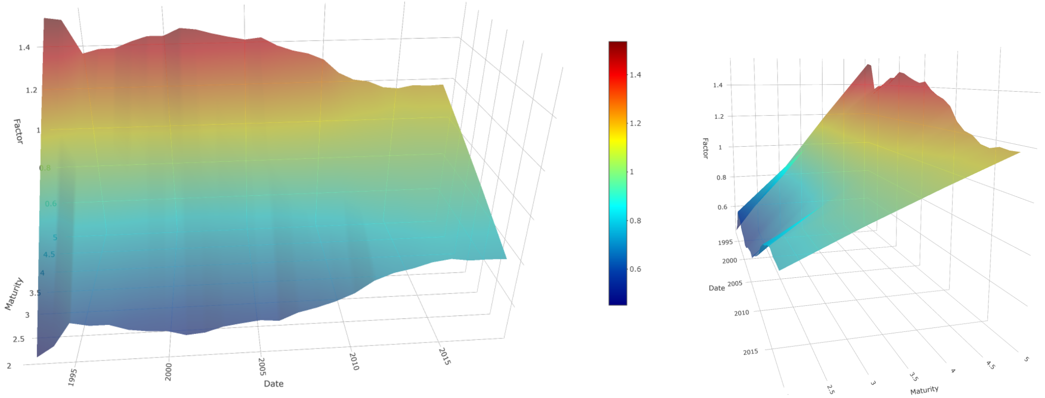
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$(\boldsymbol{\tau}^{\top}\widehat{\mathfrak{F}})_t^h$	1.268***	1.258***	1.247***	1.263***	0.511	0.626	0.736*	0.834**
	(0.315)	(0.305)	(0.334)	(0.318)	(0.422)	(0.401)	(0.409)	(0.400)
$\boldsymbol{M}_{\boldsymbol{\tau}^{\top}\widehat{\mathfrak{F}}}(\boldsymbol{\kappa}^{\top}\widehat{\boldsymbol{\xi}})_t^{(-5),h}$		0.673**		0.872***		0.738**		0.590**
		(0.281)		(0.312)		(0.315)		(0.279)
$\widehat{LN}_t^h$	1.501***	1.337***					1.518***	1.386***
	(0.421)	(0.381)					(0.387)	(0.360)
$fs_t^{(n,h)}$			0.633	0.789			0.739	0.848
			(0.698)	(0.656)			(0.658)	(0.632)
$\widehat{CP}_t^h$					1.064***	0.882**	0.967***	0.818**
					(0.380)	(0.352)	(0.343)	(0.337)
Constant	0.005	0.005	−0.116	−0.147	−0.106	−0.086	−0.248	−0.253*
	(0.111)	(0.109)	(0.166)	(0.158)	(0.117)	(0.115)	(0.158)	(0.152)
Observations	300	300	300	300	300	300	300	300
Adjusted R^2	0.082	0.098	0.031	0.062	0.054	0.074	0.103	0.114

Note: $^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$

Empirical Results

Regression Coefficients of $\left(\boldsymbol{\tau}^{\top}\widehat{\mathfrak{F}}_t\right)_t^h$ Over Time as a Function of Maturity (n)

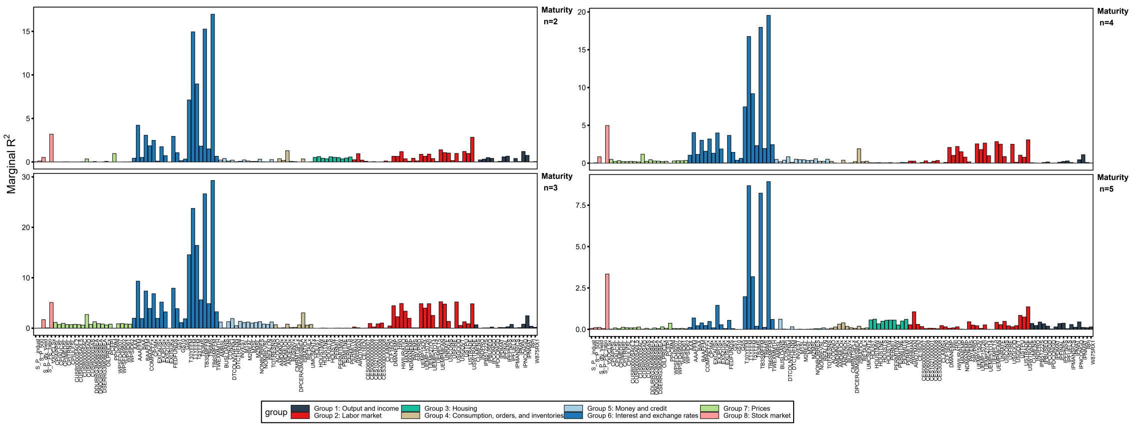
Return



Empirical Results - Economic Interpretation

Marginal R^2 of the factors $\boldsymbol{M}_{\boldsymbol{\tau}^\top\widehat{\mathfrak{F}}}(\boldsymbol{\kappa}^\top\widehat{\boldsymbol{\xi}})^{(-n),h}_{t+h/12}$

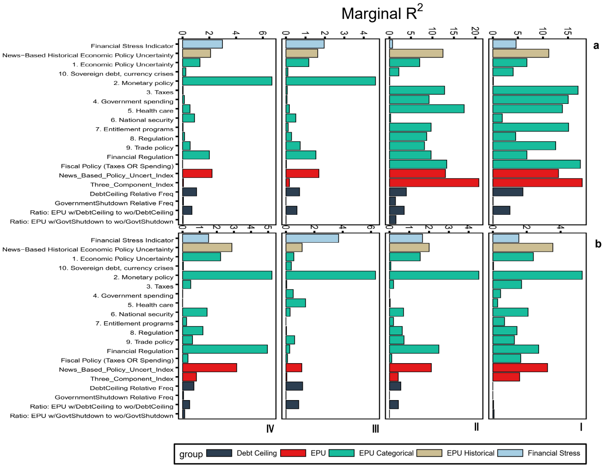
Return



Empirical Results - Economic Interpretation

Marginal R^2 Using Sentiment-Based Measures

Return



Empirical Results

Regression Coefficients of $\boldsymbol{M}_{\boldsymbol{\tau}^\top \widehat{\mathfrak{F}}}(\boldsymbol{\kappa}^\top \widehat{\xi})^{(-n),h}_{t+h/12}$ Over Time as a Function of Maturity (n)

Return